

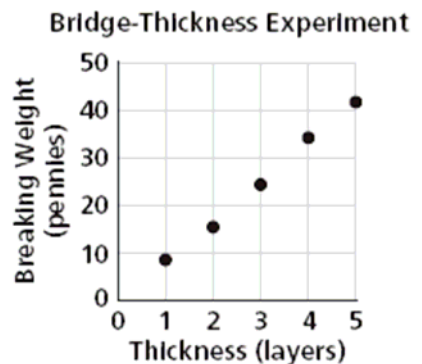
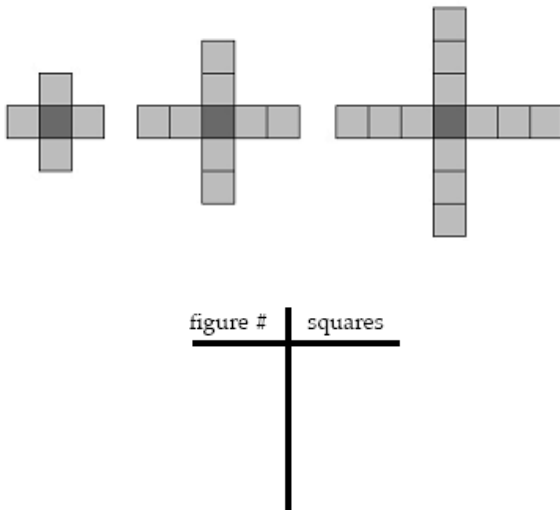
Linear Patterns

TEKS

- A.1 Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.
- (B) The student is expected to gather and record data and use data sets to determine functional relationships between quantities
- A.3 Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.
- (A) The student is expected to use symbols to represent unknowns and variables.
- (B) The student is expected to look for patterns and represent generalizations algebraically.

TAKS™

- Objective 1 The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.
- Objective 2 The student uses the properties and attributes of functions.





Key Questions

By the end of this lesson, students should be able to answer these key questions:

- How can patterns be used to develop an understanding of linear functions?
- What are the connections between a pattern model, a table, a graph, a verbal description, and a rule for the function represented in a pattern?
- What informal solution methods are useful when analyzing patterns?
- What informal solution methods developed to describe and analyze linear patterns can be connected to solving equations?

Vocabulary Focus

Constant
Expression
Linear function
Model
Pattern
Regression
Rule
Term
Variable

Students are not expected to write and define vocabulary words. Instead, these terms should be clarified as necessary in the Explain phase of the lesson. Then, they may be displayed on a word wall or recorded in a student interactive notebook.



Materials

Prepare in advance:

- ❑ Overhead transparent counters (3 red and 12 blue)
- ❑ Half sheets of grid chart paper for each group
- ❑ Overhead calculator or SmartView or TI presenter
- ❑ **Group Activity: Tile Pattern Team Challenge**
- ❑ Copy of **Team Roles**
- ❑ Color tiles or cardstock squares to model patterns

For each student:

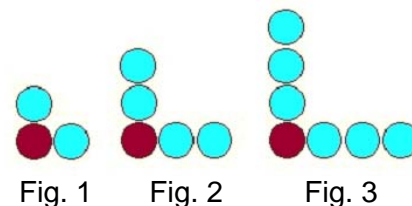
- ❑ Graphing Calculators
- ❑ **Activity: Comparing Patterns**
- ❑ **Activity: Linear Increasing and Decreasing Patterns**
- ❑ **Evaluate: Linear Patterns**

ENGAGE



The Engage portion of the lesson is designed to create student interest in the concept of studying patterns to develop generalizations that describe linear functions.

1. Display the pattern to the right on the overhead, board or chart paper. Ask students to study the pattern quietly for two minutes and write down what they see in this pattern. Encourage students to describe the pattern in words (e.g. *Figure 1 contains an arrangement of 1 dark circle with two light circles, one above and one to the right of the dark circle*).
2. Explain how these types of patterns will be used to make sense of functions. In the Explore phase of this lesson students will compare and contrast different patterns to determine how a rule can be used to describe any figure in a pattern.



Facilitation Questions

- **How can you describe the arrangement of the circles?**
Figure 1 contains an arrangement of 1 dark circle with two light circles, one above and one to the right of the dark circle.
- **How does your description of the number of circles relate to the number of the figure?**
The light circles are arranged above and to the right of the one dark circle. The number of light circles is two times the figure number plus one additional dark circle.
- **What is the difference in the total number of circles from one figure to the next?**
Figure 2 has 2 more circles than Figure 1 and Figure 3 has 2 more circles than Figure 2.
- **Do you think that this pattern represents a linear relationship? Why or why not?**
This pattern represents a linear relationship because the change in the number of circles from one figure to the next is always 2 (or constant, or the same).

EXPLORE



The Explore portion of the lesson provides students with an opportunity to be actively engaged in finding similarities and differences between linear patterns and patterns in perimeters of regular polygons. Students should identify the general characteristics of linear patterns and use these characteristics to make predictions and generalizations.

1. Distribute the **Comparing Patterns** activity page to students.
2. Allow students to work in pairs or small groups to discuss and compare the patterns. Each student in the group should be able to describe the similarities and differences between the sets of patterns.
3. Encourage students to describe the patterns in more than one way. Students should be able to identify how the pattern grows and connect it to the model. They should notice that the numerical sequences for the linear patterns grow by a constant amount.
4. Students may struggle to create a pattern that fits the numerical sequence. Provide toothpicks, tiles, or counters to help students build and draw their pattern. Encourage groups to create slightly different models.

Facilitation Questions

- **Are there any patterns in Set A that have the same numerical values for each of the figures?**
There 3, 5, and 7 objects in both the toothpick pattern and the circles pattern.
- **How is the tiles pattern in Set A different from the other two patterns?**
The tile pattern has 5, 9, and 13 tiles in each figure.
- **What number is added to each figure in Set A to make the next figure?**
In the tile pattern each figure increases by 4 tiles. In the toothpick and circles patterns each figure increases by 2.
- **How do the figures in Set B increase?**
The perimeters of the triangles increase by 1 for each figure. The perimeters of the squares increase by 2 for each figure. The perimeters of the pentagons increase by 3 for each figure.
- **How are the numerical sequences for the perimeters for each set of regular polygons similar or different?**
Each sequence begins with a different amount and increases by a different amount (1 more than the previous polygon).
- **How can you use the amount of increase (or decrease) to write a rule or generalization for a pattern?**
The amount of the decrease is related to what must be multiplied by x to find the rule for a pattern. The amount that you start with or the part added can be found by subtracting the amount of the increase or decrease from the number of objects in the first figure.

Name _____ Period _____ Date _____

Comparing Patterns

How are the following patterns alike? How are they different? Write two or three sentences that describe the similarities and differences between these three patterns.

SET A

Each of the figures in Set B is made of regular polygons. Consider one side of a polygon as one unit to calculate the perimeter in units. How is the perimeter of the following patterns alike? How are they different? Write two or three sentences that describe the similarities and differences between these three patterns.

SET B

Name _____ Period _____ Date _____

Compare the patterns in Set A with the patterns in Set B and write at least three sentences describing how the patterns are similar and different pictorially and numerically.

What is the same and what is different about these two number sequences?

Sequence A
2 5 10 17 26...

Sequence B
2 5 8 11 14...

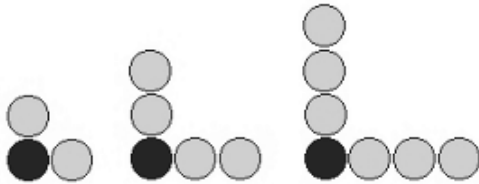
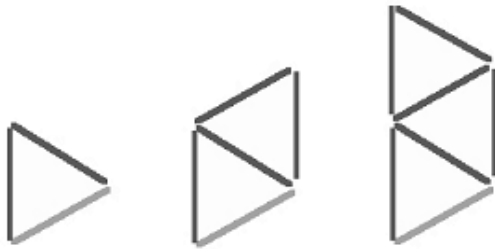
Work with a group to create a model of a quadratic growing pattern that fits this numerical sequence.

3 7 13 21 31 43...

Comparing Patterns

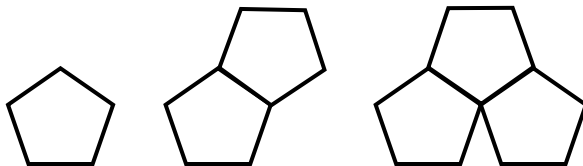
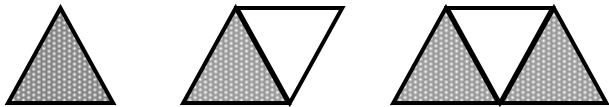
How are the following patterns alike? How are they different? Write two or three sentences that describe the similarities and differences between these three patterns.

SET A



Each of the figures in **Set B** is made of regular polygons. Consider one side of a polygon as one unit to calculate the perimeter in units. Find the perimeter for each figure in the patterns. How is the perimeter of the following patterns alike? How are they different? Write two or three sentences that describe the similarities and differences between these three patterns.

SET B



Name: _____ Period: _____ Date: _____

Compare the patterns in **SET A** with the patterns in **SET B** on the previous page and write at least three sentences describing how the patterns are similar and different pictorially and numerically.

What is the same and what is different about these two number sequences?

Sequence A

2 4 6 8 10...

Sequence B

3 5 7 9 11...

Work with a group to create a model of a linear growing pattern that fits this numerical sequence.

5 9 13 17 21 25...

EXPLAIN



The Explain portion of the lesson provides students with an opportunity to use algebraic reasoning to connect multiple representations to models of linear growing patterns.

1. Distribute one page of the **Tile Pattern Team Challenge** activity sheet, the **Team Roles** sheet and graphing calculators to each group of four students. Two groups may have to work on the same pattern if there are more than six groups.
2. Review the team roles with students and assign each member of the group a role. Set criteria for the group posters and presentations. Consider giving each member of a group a different colored marker and ask that they use only that color for their work on the poster.
3. Monitor students as they work in groups so that each member is included and understands that they will have a part in the final presentations. Allow each group 3-5 minutes for their poster presentations.
4. Once all groups have presented lead a whole class discussion about using the graphing calculator, lists, and linear regression to check the rules and posters.
5. Post the group posters around the room and have students give peer feedback using the “*noticings*” and “*wonderings*” protocol. Set criteria for the feedback and ask students to identify similarities and differences between the different patterns.

Facilitation Questions

- **How can you use different representations to help discover the rule for the pattern?**
Answers will vary.
- **How are the different representations connected? How does the table connect to the graph and the rule? How do the graph and the rule connect to the table?**
Answers will vary.
- **Which representation was the most helpful for finding the rule that would work for any figure in the pattern?**
Answers will vary.

Team Roles

Resource Manager:

- Call the teacher over when your team is stuck. Make sure that all questions are team questions.
“What team question can we ask the teacher?”
“Are we sure that no one here can answer the question?”
- Don't let your team stay stuck!

Facilitator:

- Make sure your team understands the entire task before you begin.
“Who wants to read? Does everyone understand what we are asked to do?”
“What is the connection? How will it show in the graph? How will it show in the table?”
- Keep your team together. Make sure everyone's ideas are heard.
“Are we all ready to move onto the next step?”
“Does anyone see it in a different way?”

Recorder/Reporter:

- Help your team organize a transparency or poster with your analysis. Your poster needs to show everyone's ideas and be well organized. Use color, arrows, and other math tools to communicate your mathematics, reasons and connections.
“How can we show that on the graph?”
“How can we show that connection?”

Task Manager:

- Be sure that your team is accomplishing the task effectively and efficiently. Keep track of the time and tell the team when it is time to move forward to the next part of the task. Make sure that all talking is within your team and is helping you accomplish the task. Eliminate side conversations.
“How can we divide the work most efficiently?”
“We need to finish this part in 5 minutes, so we have time for...”

Tile Pattern Team Challenge

Your team's task is to create a poster showing every way you can represent the pattern below and highlighting all of the connections between the representations that you can find. For this activity, finding and showing the connections are the most important parts. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.

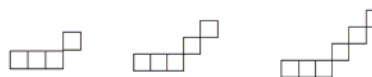


Figure 1

Figure 2

Figure 3

Pattern Analysis:

- **Extend the pattern:** Draw Figures 0, 4, and 5. Then describe Figure 20. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- **Generalize the pattern** by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What **connections** do you see between the different representations (graph, figures, and table)? How can you show these connections?

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 20. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

Team Roles

Resource Manager:

- Call the teacher over when your team is stuck. Make sure that all questions are team questions.

“What team question can we ask the teacher?”

“Are we sure that no one here can answer the question?”

- Don't let your team stay stuck!

Facilitator:

- Make sure your team understands the entire task before you begin.

“Who wants to read? Does everyone understand what we are asked to do?”

“What is the connection? How will it show in the graph? How will it show in the table?”

- Keep your team together. Make sure everyone's ideas are heard.

“Are we all ready to move onto the next step?”

“Does anyone see it in a different way?”

Recorder/Reporter:

- Help your team organize a transparency or poster with your analysis. Your poster needs to show everyone's ideas and be well organized. Use color, arrows, and other math tools to communicate your mathematics, reasons and connections.

“How can we show that on the graph?”

“How can we show that connection?”

Task Manager:

- Be sure that your team is accomplishing the task effectively and efficiently. Keep track of the time and tell the team when it is time to move forward to the next part of the task. Make sure that all talking is within your team and is helping you accomplish the task. Eliminate side conversations.

“How can we divide the work most efficiently?”

“We need to finish this part in 5 minutes, so we have time for...”

Tile Pattern Team Challenge

Your team's task is to create a poster showing every way you can represent the pattern below and highlighting all of the connections between the representations that you can find. For this activity, **finding and showing the connections are the most important parts**. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.



Figure 1

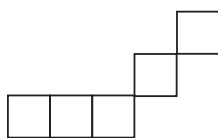


Figure 2

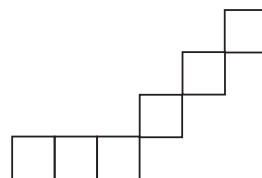


Figure 3

Pattern Analysis:

- **Extend** the pattern: Draw Figures 0, 4, and 5. Then describe Figure 20. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- **Generalize** the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What **connections** do you see between the different representations (graph, figures, and table)? How can you show these connections?

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 20. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

Tile Pattern Team Challenge

Your team's task is to create a poster showing every way you can represent the pattern below and highlighting all of the connections between the representations that you can find. For this activity, **finding and showing the connections are the most important parts**. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.

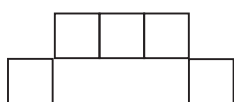


Figure 1

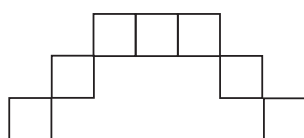


Figure 2

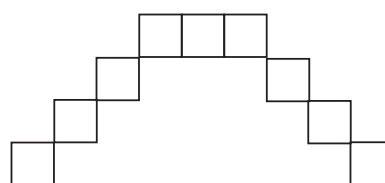


Figure 3

Pattern Analysis:

- **Extend** the pattern: Draw Figures 0, 4, and 5. Then describe Figure 20. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- **Generalize** the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What **connections** do you see between the different representations (graph, figures, and table)? How can you show these connections?

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 20. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

Tile Pattern Team Challenge

Your team's task is to create a poster showing every way you can represent the pattern below and highlighting all of the connections between the representations that you can find. For this activity, **finding and showing the connections are the most important parts**. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.

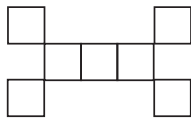


Figure 1

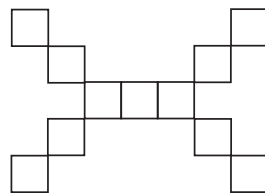


Figure 2

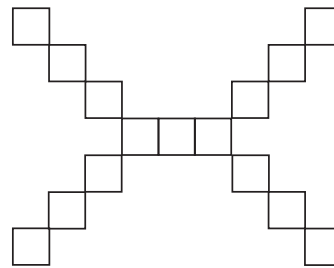


Figure 3

Pattern Analysis:

- **Extend** the pattern: Draw Figures 0, 4, and 5. Then describe Figure 20. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- **Generalize** the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What **connections** do you see between the different representations (graph, figures, and table)? How can you show these connections?

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 20. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

Tile Pattern Team Challenge

Your team's task is to create a poster showing every way you can represent the pattern below and highlighting all of the connections between the representations that you can find. For this activity, **finding and showing the connections are the most important parts**. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.

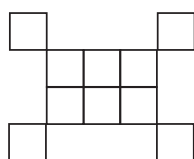


Figure 1

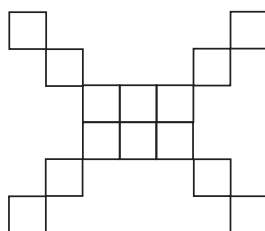


Figure 2

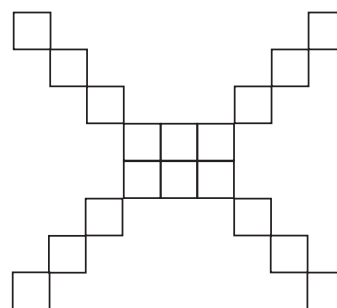


Figure 3

Pattern Analysis:

- **Extend** the pattern: Draw Figures 0, 4, and 5. Then describe Figure 20. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- **Generalize** the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What **connections** do you see between the different representations (graph, figures, and table)? How can you show these connections?

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 20. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

Tile Pattern Team Challenge

Your team's task is to create a poster showing every way you can represent the pattern below and highlighting all of the connections between the representations that you can find. For this activity, **finding and showing the connections are the most important parts**. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.

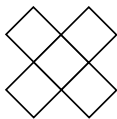


Figure 1

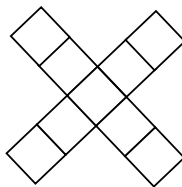


Figure 2

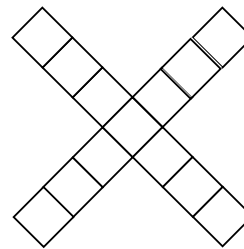


Figure 3

Pattern Analysis:

- **Extend** the pattern: Draw Figures 0, 4, and 5. Then describe Figure 20. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- **Generalize** the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What **connections** do you see between the different representations (graph, figures, and table)? How can you show these connections?

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 20. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

Tile Pattern Team Challenge

Your team's task is to create a poster showing every way you can represent the pattern below and highlighting all of the connections between the representations that you can find. For this activity, **finding and showing the connections are the most important parts**. Clearly presenting the connections between representations on your poster will help you convince your classmates that your description of the pattern makes sense.

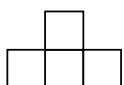


Figure 1

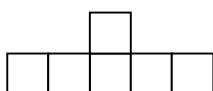


Figure 2

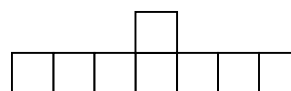


Figure 3

Pattern Analysis:

- **Extend** the pattern: Draw Figures 0, 4, and 5. Then describe Figure 20. Give as much information as you can. What will it look like? How will the tiles be arranged? How many tiles will it have?
- **Generalize** the pattern by writing a rule that will give the number of tiles in any figure in the pattern. Show how you got your answer.
- Find the number of tiles in each figure. Record your data in a table and a graph.
- Demonstrate how the pattern grows using color, arrows, labels, and other math tools to help you show and explain. Show growth in each representation.
- What **connections** do you see between the different representations (graph, figures, and table)? How can you show these connections?

Presenting the Connections:

As a team, organize your work into a large poster that clearly shows each representation of your pattern, as well as a description of Figure 20. When your team presents your poster to the class, you will need to support each statement with a reason from your observations. Each team member must explain something mathematical as part of your presentation.

ELABORATE



The Elaborate portion of the lesson provides students with an opportunity to make generalizations about linear patterns. Students are provided an example of how different algebraic expressions can be used to describe the same model and should explain how the different expressions are equivalent. This part of the lesson is designed for individual investigation.

1. Distribute the **Linear Increasing and Decreasing Patterns** activity sheet. Students can cut up the page and attach it to a page in their interactive notebook. Give students graph paper if they request it. Otherwise, encourage them to sketch their calculator screens.
2. Allow students to use whatever tools they have used previously such as a graphing calculator (and including integer pieces or colored tiles) to complete this activity. Monitor student progress and address any misconceptions. Use the facilitation questions to assist struggling students.
3. If students use the graphing calculator to find the rule for the patterns, encourage them to connect their rule to the pattern and other representations.
4. For Part 2, students should describe the concept of equivalence using words, pictures, concrete models, or algebraic methods. If students enter each expression in the graphing calculator, they must be able to justify how this strategy works.

Name: _____ Period: _____ Date: _____

Linear Increasing and Decreasing Patterns

Part 1
For each of the patterns below draw the 4th and 5th figures of the sequence. Describe the 20th figure using words, pictures and/or symbols. How would you find any figure in the sequence? Create a table, graph and rule for each pattern. (You may cut out the patterns and tape them in your spiral notebook.)

Pattern 1
Figure 1: 2 squares (1x2)
Figure 2: 3 squares (1x3)
Figure 3: 4 squares (1x4)

Pattern 2
Figure 1: 3 squares (2x2)
Figure 2: 5 squares (2x3)
Figure 3: 7 squares (2x4)

Pattern 3
Figure 1: 10 squares (2x5)
Figure 2: 15 squares (3x5)
Figure 3: 20 squares (4x5)

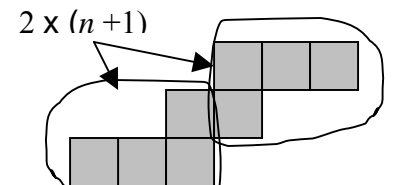
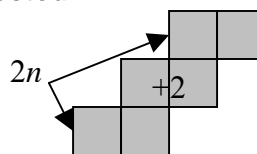
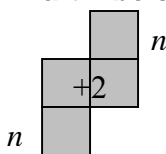
Part 2

Which of these algebraic expressions could be linked to the growing pattern above? Demonstrate how one or more of the expressions links to the pattern using color, arrows, labels, and other math tools to help you show and explain.

A. $2n + 2$
B. $n + 2 + n$
C. $2(n+1)$

Facilitation Questions

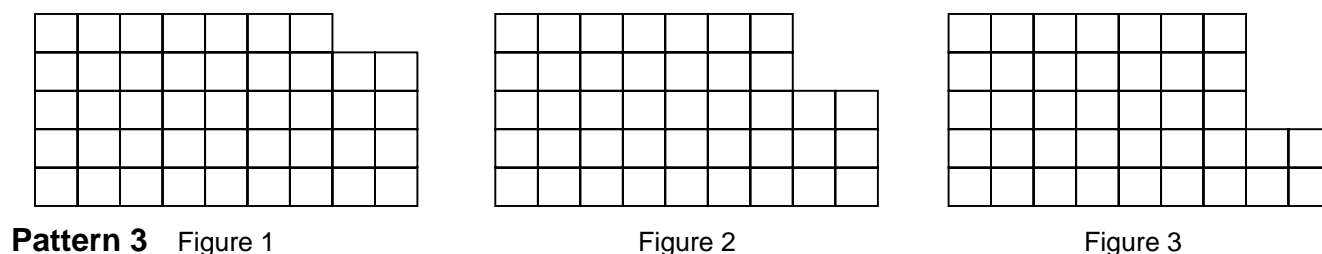
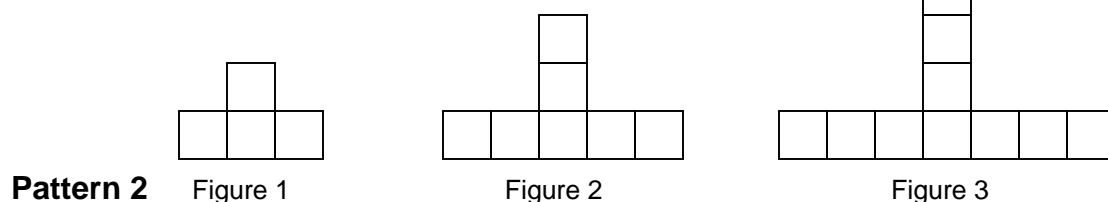
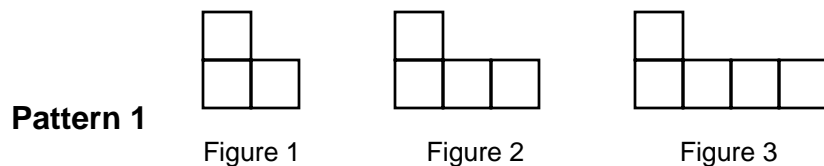
- **How can you describe each figure in the sequence in relation to the figure number?**
Answers will vary, but some variation of an expression may emerge.
- **How can you use a table and/or graph to find the rule for the pattern?**
Students might describe how to create a table to find the rate of change and starting point for the pattern.
- **What information about the pattern can be found by graphing?**
Students may be able to recall previous learning about how to use slope and y-intercept to write the rule for the pattern.
- **How can you use the graphing calculator to check your work?**
Answers will vary.
- **How can the expressions in Part 2 be connected to the model?**
Answers will vary.



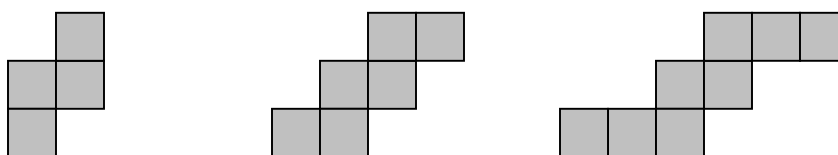
Linear Increasing and Decreasing Patterns

Part 1

For each of the patterns below draw the 4th and 5th figures of the sequence. Describe the 20th figure using words, pictures and/or symbols. How would you find any figure in the sequence? Create a table, graph and rule for each pattern. (You may cut out the patterns and tape them in your spiral notebook.)



Part 2



Which of these algebraic expressions could be linked to the growing pattern above? Demonstrate how one or more of the expressions links to the pattern using color, arrows, labels, and other math tools to help you show and explain.

A $2n + 2$

B $n + 2 + n$

C $2(n+1)$

EVALUATE



During the Evaluate portion of the lesson, teachers will assess student learning about the concepts and procedures that were investigated and developed during the lesson.

1. Distribute a copy of **Evaluate: Linear Patterns** to each student.
2. Ask the students to answer the **Evaluate** questions.
3. The error analysis provided below guides teachers in diagnosing student misunderstandings so that the teacher can more efficiently reteach the topic based on conceptual or procedural student error patterns.

Question Number	Correct Answer	TEKS	TAKS™	Conceptual Error			Procedural Error			Guess
1	A	A.3B	2				B	C	D	
2	B	A.1B	1				A	C	D	
3	C	A.3B	2				A	B		D
4	C	A.3B	2					B	D	A

Evaluate: Linear Patterns

1. The squares below show a pattern.

Stage 1	□ □
Stage 2	□ □ □ □ □ □
Stage 3	□ □ □ □ □ □ □ □ □ □
Stage 4	□ □ □ □ □ □ □ □ □ □ □ □ □ □

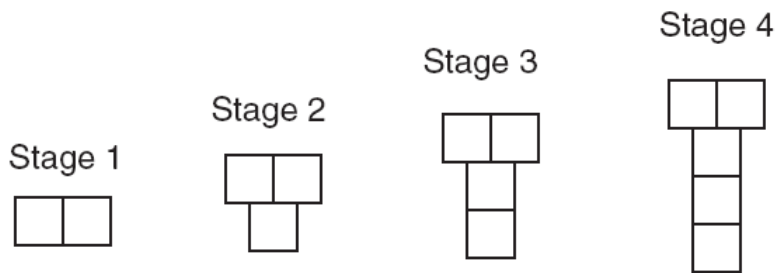
Which expression can be used to determine the number of squares at stage n ?

- A $4n - 2$
 - B $5n - 3$
 - C $n^2 + n$
 - D $2n^2$
2. Which of the following equations best represents the relationship in the set of data shown below?

x	-4	-3	-1	2	4
y	24	20	12	0	-8

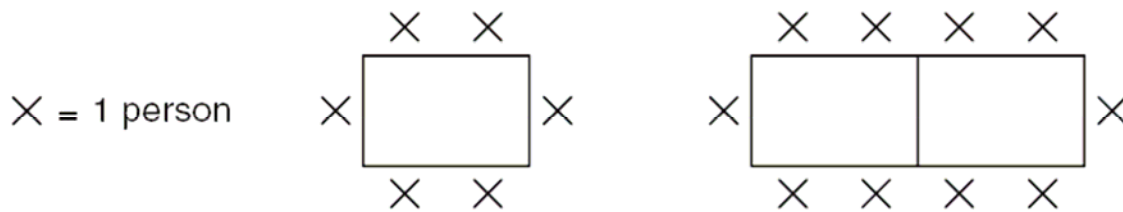
- A $y = -5x + 4$
- B $y = -4x + 8$
- C $y = -7x - 4$
- D $y = -8x - 8$

3. The blocks below are arranged in sequence to show a pattern.



Which expression can be used to determine the number of blocks at Stage n ?

- A $n + 1$
- B $3n - 1$
- C $(n - 1) + 2$
- D $(n - 1) + 2n$
4. For a sports banquet Coach Brown must use rectangular tables in the school cafeteria. The diagram below shows the seating arrangement that Coach Brown can use at 1 and 2 tables.



Which expression can be used to determine the number of people who can sit as a group if y tables are joined to form 1 long table?

- A $4(y + 1)$
- B $3(y + 1)$
- C $2(2y + 1)$
- D $6y$